

Constraints on Light Pseudoscalars Implied by Tests of the Gravitational Inverse-Square Law

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Abstract

The exchange of light pseudoscalars between fermions leads to a spin-independent potential in order g^4 , where g is the Yukawa pseudoscalar-fermion coupling constant. This potential gives rise to detectable violations of both the weak equivalence principle (WEP) and the gravitational inverse-square law (ISL), even if g is quite small. We show that when previously derived WEP constraints are combined with those arising from ISL tests, a direct experimental limit on the Yukawa coupling of light pseudoscalars to neutrons can be inferred for the first time ($g_n^2/4\pi \lesssim 1.6 \times 10^{-7}$), along with a new (and significantly improved) limit on the coupling of light pseudoscalars to protons.

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In a previous paper [1], it was shown that laboratory bounds on the Yukawa couplings of light pseudoscalars to protons and neutrons could be significantly improved by using the results from recent weak equivalence principle (WEP) experiments [2]. These experiments are sensitive to the spin-independent long-range forces that arise in order g^4 from two-pseudoscalar exchange [1,3,4], where g is defined by the coupling

$$\mathcal{L}(x) = ig\bar{\psi}(x)\gamma_5\psi(x)\phi(x). \quad (1)$$

Here $\phi(x)$ is the field operator for a pseudoscalar of mass m , and $\psi(x)$ denotes either a proton (p), electron (e), or neutron (n) of mass M_p , M_e , or M_n respectively. For each pair of interacting particles, $\mathcal{L}(x)$ leads to a potential $V^{(4)}(r)$ in order g^4 which in the $m \rightarrow 0$ limit is given by [3,4]

$$V_{ab}^{(4)}(r) = -\frac{g_a^2 g_b^2}{64\pi^3 M_a M_b} \frac{1}{r^3}, \quad (2)$$

where a and b may each denote p , e , or n . The object of the present paper is to demonstrate that already-existing data from tests of the gravitational inverse-square law (ISL) [5,6] provide new stringent constraints on g_p^2 and g_n^2 . When combined with the constraints implied by Eq. (3) below and the data from the WEP test in Ref. [2], the ISL data lead to the first direct experimental bound on the pseudoscalar-neutron coupling constant g_n^2 , and to a significantly improved bound on g_p^2 (see Eq. (10) below).

Leaving aside for the moment the contribution from electrons, it was shown in Ref. [1] that $V_{ab}^{(4)}$ leads to an acceleration difference $\Delta\vec{a}_{2-2'}$ of macroscopic test objects 2 and 2' in the presence of a common source M_1 . If these have masses M_2 and $M_{2'}$, and contain Z_2 (N_2) protons (neutrons), and $Z_{2'}$ ($N_{2'}$) protons (neutrons) respectively, then

$$\Delta\vec{a}_{2-2'} = \vec{\mathcal{F}}(\vec{r}) \left(\frac{M_1}{m_H^2} \right) \left[g_p^2 \left(\frac{Z_1}{\mu_1} \right) + g_n^2 \left(\frac{N_1}{\mu_1} \right) \right] \left[g_p^2 \Delta \left(\frac{Z}{\mu} \right)_{2-2'} + g_n^2 \Delta \left(\frac{N}{\mu} \right)_{2-2'} \right]. \quad (3)$$

In Eq. (3), $\vec{\mathcal{F}}(\vec{r})$ is the integral over the mass distribution of the source [1], $\mu_i = M_i/m_H$, $m_H = m({}_1\text{H}^1)$, $M_n \simeq M_p \equiv M$, and $\Delta(Z/\mu)_{2-2'} = Z_2/\mu_2 - Z_{2'}/\mu_{2'}$, etc., [6]. Since all the parameters appearing in Eq. (3) are known, except for the pseudoscalar couplings g_p^2 and g_n^2

(to protons and neutrons respectively), an experimental determination of $\Delta\vec{a}_{2-2'}$ leads to a constraint on g_p^2 and g_n^2 .

As noted in Ref. [1], however, the right-hand side of Eq. (3) vanishes whenever g_p^2 and g_n^2 satisfy

$$\frac{g_p^2}{g_n^2} = -\frac{\Delta(N/\mu)_{2-2'}}{\Delta(Z/\mu)_{2-2'}}, \quad (4)$$

in which case g_p^2 and g_n^2 can be arbitrarily large and still be compatible with any experimental bound on $\Delta\vec{a}_{2-2'}$. Since the right-hand side of Eq. (4) is close to 1 for most pairs of materials, including those used in Ref. [2], Eq. (4) can be satisfied even when g_p^2 and g_n^2 are each quite large provided $g_p^2 \simeq g_n^2$. This is shown graphically in Fig. 1 which plots the constraints in the g_p^2 - g_n^2 plane that emerge when Eq. (3) is combined with the experimental limits of Gundlach, *et al.* [2]. It is seen that the boundary of the allowed region is a hyperbola with an asymptote near $g_p^2 = g_n^2$, along which no limits on g_p^2 or g_n^2 can be inferred. To circumvent the problem caused by such “hyperbolic” constraints, one can combine results from experiments using different materials, which thus have slightly different asymptotes. Alternatively one can choose special materials (such as $2 = \text{Li}$ and $2' = \text{Ru}$) for which Eq. (4) can never hold [1], and which thus lead to ellipses in the g_p^2 - g_n^2 plane. The combination of “elliptical” and “hyperbolic” constraints would then lead to separate bounds on g_p^2 and g_n^2 . As we now demonstrate, existing data from ISL tests also provide elliptical constraints on g_p^2 and g_n^2 and, when combined with earlier WEP results, lead directly to the bounds quoted in Eq. (10) below.

Consider the ISL experiment of Spero, *et al.* [5,6] in which a cylindrical Cu test mass is suspended at the end of a torsion fiber inside a larger hollow stainless steel cylinder. It can be shown that for infinitely long cylinders the Cu test mass will experience a force from the stainless steel cylinder only if the underlying interaction is not a pure $1/r^2$ force. When the small (and calculable) end effects due to finite cylinders are taken into account, the experiment of Spero, *et al.* becomes a null test for the presence of new non-Newtonian inverse-power-law interactions, such as $V_{ab}^{(4)}(r)$ in Eq. (2). We will use two convenient pa-

parameterizations of power law potentials between two particles 1 and 2 [7,8]:

$$V_n(r) = -\alpha_n \left(\frac{G_N M_1 M_2}{r} \right) \left(\frac{r_0}{r} \right)^{n-1}, \quad (5a)$$

$$= -\Lambda_n \left(\frac{B_1 B_2}{r} \right) \left(\frac{r_0}{r} \right)^{n-1}. \quad (5b)$$

Here G_N is the Newtonian gravitational constant, $r_0 = 1$ fm is an arbitrarily chosen length scale, B_1 and B_2 are the baryon numbers for bodies 1 and 2 respectively, and Λ_n and α_n are dimensionless constants characterizing the strength of the interaction. When gravity is included, the total potential energy between these two point masses is given by

$$V_{\text{tot}}(r) = -\frac{G_N M_1 M_2}{r} \left[1 + \alpha_n \left(\frac{r_0}{r} \right)^{n-1} \right], \quad (6a)$$

$$= -\frac{G_N M_1 M_2}{r} \left[1 + \Lambda_n \left(\frac{m_P}{m_H} \right)^2 \left(\frac{B_1}{\mu_1} \right) \left(\frac{B_2}{\mu_2} \right) \left(\frac{r_0}{r} \right)^{n-1} \right], \quad (6b)$$

where $m_P \equiv \sqrt{\hbar c / G_N}$ is the Planck mass. The null results of the ISL test of Spero, *et al.* can then be used to set limits on α_n or Λ_n , after integrating the corresponding $1/r^{n+1}$ force laws over the mass distributions of the Cu test mass and the stainless steel cylinder [9]. The 1σ limits implied by Spero, *et al.* for α_n and Λ_n are shown in Table I for several physically relevant values of n . The results in Table I, which were obtained by direct integration over the mass distributions of the interacting Cu and stainless steel cylinders, are in excellent agreement with those obtained previously by Mostepanenko and Sokolov [10] who used a phenomenological parameterization of the non-Newtonian interaction to constrain Λ_n . Although we are specifically concerned with the case $n = 3$, other values of n are also interesting: $n = 2$ potentials can arise from 2-scalar exchange, as well as 2-photon exchange [11], and $n = 5$ characterizes the 2-body potential from neutrino-antineutrino exchange [12] and the 2-pseudoscalar exchange potential with derivative coupling (which is applicable to axions) [4]. Note, however, that $n = 1$ is uninteresting since such a potential would not lead to a deviation from the inverse-square law, but only to a modified value of G_N (which would be difficult to detect). Table I also presents the 1σ limits derived from the experiment of Mitrofanov and Ponomareva (MP) [13] which is a test of the ISL over the

range 3.8–6.5 mm. In this experiment a modified Cavendish apparatus is used to measure the force between a mass A suspended at one end of a torsion balance, and a second mass B whose distance from A is varied. The experiment then compares the experimental value for the force ratio $F(r_1)/F(r_2)$, where $r_1 = 3.773(40)$ mm and $r_2 = 6.473(40)$ mm, to the calculated ratio expected assuming Newtonian gravity. We see from Table I that for the case $n = 3$, which is our concern in this paper, the limits implied by Spero, *et al.* are more stringent than those of MP, although for $n = 4, 5, 6$ the reverse is true.

To extract constraints on the pseudoscalar coupling constants g_p^2 and g_n^2 from Spero, *et al.*, we begin by considering the interaction of two macroscopic objects separated by a distance r that is large compared to their dimensions. From Eq. (2) the total 2-body interaction energy $V_{12}^{(4)}$ in order g^4 is given by

$$V_{12}^{(4)}(r) = -\frac{1}{64\pi^3 M^2} \frac{1}{r^3} \left(g_p^2 Z_1 + g_n^2 N_1 \right) \left(g_p^2 Z_2 + g_n^2 N_2 \right), \quad (7)$$

which should be compared with the parameterizations of Eq. (5) for $n = 3$. For the actual geometry of the Spero experiment one must integrate over the mass distributions of the inner test mass and the outer cylinder [9], so that $1/r^3$ in Eqs. (7) and (5) is replaced by the appropriate average $\langle 1/r^3 \rangle$. By combining Eqs. (7) and (5) for $n = 3$ the constraint implied by Spero, *et al.* can be expressed in the form

$$\left(g_p^2 \frac{Z_1}{\mu_1} + g_n^2 \frac{N_1}{\mu_1} \right) \left(g_p^2 \frac{Z_2}{\mu_2} + g_n^2 \frac{N_2}{\mu_2} \right) = 64\pi^3 G_N M^2 m_H^2 \alpha_3 r_0^2. \quad (8)$$

Using the 1σ limits from Spero, *et al.* presented in Table I we then find

$$\left(0.469g_p^2 + 0.540g_n^2 \right) \left(0.460g_p^2 + 0.549g_n^2 \right) \lesssim 3.5 \times 10^{-12}. \quad (9)$$

In Eq. (9) the expression in the first set of parentheses arises from the hollow cylinder, where the coefficients of g_p^2 and g_n^2 are the values of Z/μ and N/μ , respectively, for stainless steel. Similarly, the expression in the second set of parentheses represents the contributions from the Cu test mass, while the right hand side of Eq. (9) is derived from the bound on $\alpha_3 r_0^2$ quoted in Table I.

We see immediately from Eq. (9) that the constraint implied by the ISL experiment of Spero, *et al.* [5] leads to an ellipse in the g_p^2 - g_n^2 plane, as can be seen in Fig. 1. This is, of course, related to the fact that the left side of Eq. (9) cannot vanish unless both g_p^2 and g_n^2 do. Figure 1 also exhibits the previously derived WEP constraint [1] from the experiment of Gundlach, *et al.* [2], which gives rise to a hyperbola in the g_p^2 - g_n^2 plane as we have noted previously. The significant new feature of Fig. 1 is that the combination of the hyperbolic WEP constraint and the elliptical ISL constraint lead to upper bounds on g_p^2 and g_n^2 separately. We find from the figure the following 1σ limits:

$$g_p^2/4\pi \lesssim 1.6 \times 10^{-7}, \quad (10a)$$

$$g_n^2/4\pi \lesssim 1.6 \times 10^{-7}. \quad (10b)$$

The result for g_n^2 in Eq. (10) represents the first direct laboratory constraint on the Yukawa coupling of pseudoscalars to neutrons. We note that the only previous laboratory limit on g_n^2 [1] was based on an indirect model-dependent argument due to Daniels and Ni [14] utilizing the spin-dependent results of Ritter, *et al.* [15]. For g_p^2 there is an earlier result due to Ramsey [16], $g_p^2/4\pi \lesssim 2.5 \times 10^{-5}$ (1σ), which was obtained from a study of the molecular spectrum of H_2 . As we see from Eq. (10), the limit implied by combining the ISL and WEP results improves the Ramsey limit by more than two orders of magnitude.

We can also extract from Fig. 1 constraints on g_p^2 and g_n^2 in special cases of interest. For a universal coupling to baryon number $g_p^2 = g_n^2$, and we find (at the 1σ level)

$$g_{p,n}^2/4\pi \lesssim 1.5 \times 10^{-7}. \quad (11)$$

The similarity of the results of Eqs. (10) and (11) arises because the largest allowed values of g_p^2 and g_n^2 lie near the line $g_p^2 = g_n^2$, as can be seen in Fig. 1. Two other results of interest are the limiting cases $g_p^2 \gg g_n^2$ and $g_n^2 \gg g_p^2$. We can see from Fig. 1 that these are determined by the WEP results of Gundlach, *et al.*, and hence can be taken over from our previous analysis [1]:

$$g_p^2/4\pi \lesssim 9 \times 10^{-8}, \quad (g_p^2 \gg g_n^2), \quad (12a)$$

$$g_n^2/4\pi \lesssim 7 \times 10^{-8}, \quad (g_n^2 \gg g_p^2). \quad (12b)$$

Although we have focused thus far on the pseudoscalar couplings to protons and neutrons, it is straightforward to show that the contributions from electrons can be incorporated via the substitution $g_p^2 \rightarrow g_c^2 \equiv g_p^2 + (M/M_e)g_e^2$, where g_e is the pseudoscalar-electron coupling constant. This follows from Eq. (2) by noting that the contributions from electrons are enhanced by a factor M/M_e relative to those from protons and neutrons. The limits on $g_p^2/4\pi$ in Eqs. (8)–(12) can then be taken over immediately for $g_c^2/4\pi$, and these lead to constraints on g_e^2 , at least in principle. In practice, however, existing limits on $g_e^2/4\pi$ obtained from spin-dependent experiments [17] are more stringent, $g_e^2/4\pi \lesssim 10^{-16}$. It follows that despite the enhancement arising from the factor M/M_e , the contribution from the term in $g_c^2/4\pi$ proportional to g_e^2 is at most of order 10^{-13} . Thus, the bounds on $g_e^2/4\pi$ implied by Eqs. (8)–(12) are in fact bounds on $g_p^2/4\pi$, and the prospects for constraining g_e^2 via $V_{ab}^{(4)}$ seem quite remote at present.

The limits on g_p^2 and g_n^2 in Eqs. (9)–(12) are the most restrictive direct laboratory constraints currently available. Although astrophysical arguments based on stellar cooling calculations are more stringent [18], typically $g_{p,n}^2/4\pi \lesssim 10^{-21}$, they are necessarily more model dependent. For derivative-coupled pseudoscalars such as axions, there is at present no viable alternative to astrophysical bounds, since those arising from existing laboratory experiments (corresponding to $n = 5$ in Table I) are too weak to be of use. However, by adapting the present formalism future laboratory experiments carried out over shorter distance scales may give rise to useful bounds on axions, as we will discuss in more detail elsewhere.

In summary, we have shown that the limits implied by the ISL experiment of Spero, *et al.* [5], complement those previously derived from the WEP experiment of Gundlach, *et al.* [2], and together allow the pseudoscalar-neutron coupling constant g_n^2 to be directly determined for the first time. In addition, the combination of these two experiments leads to a new bound on the pseudoscalar-proton coupling constant g_p^2 , which improves on the earlier Ramsey limit

[16] by more than two orders of magnitude. As was noted in Ref. [1], the Gundlach results lead to hyperbolic constraints on g_p^2 and g_n^2 which admit the possibility that each of these could be quite large, provided that $g_p^2 \simeq g_n^2$. Absolute bounds on these constants could be obtained, however, by using special materials such as Li and Ru. What we have shown here is that *already existing* data from the ISL experiment of Spero, *et al.* provide the needed elliptical constraints, in effect playing the same role that a WEP experiment utilizing Li and Ru would. The fact that the WEP and ISL experiments complement each other in this way raises the possibility of a new generation of WEP and ISL experiments whose results, when combined, would lead to even more stringent constraints on g_p^2 and g_n^2 . Although the laboratory constraints may not be as restrictive as those implied by astrophysical limits [18], they are completely model-independent, and furthermore allow g_p^2 and g_n^2 to be separately determined.

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TABLES

| n | Spero, <i>et al.</i> | | Mitrofanov and Ponomareva | |
|-----|-----------------------------------|-----------------------|-----------------------------------|-----------------------|
| | $\alpha_n r_0^{n-1}$ | Λ_n | $\alpha_n r_0^{n-1}$ | Λ_n |
| 2 | $1.3 \times 10^{-6} \text{ m}$ | 7.7×10^{-30} | $6.8 \times 10^{-5} \text{ m}$ | 4.0×10^{-28} |
| 3 | $1.3 \times 10^{-8} \text{ m}^2$ | 7.7×10^{-17} | $8.1 \times 10^{-8} \text{ m}^2$ | 4.7×10^{-16} |
| 4 | $1.7 \times 10^{-10} \text{ m}^3$ | 9.9×10^{-4} | $1.3 \times 10^{-10} \text{ m}^3$ | 7.5×10^{-4} |
| 5 | $2.3 \times 10^{-12} \text{ m}^4$ | 1.4×10^{10} | $2.1 \times 10^{-13} \text{ m}^4$ | 1.2×10^9 |
| 6 | $3.2 \times 10^{-14} \text{ m}^5$ | 1.8×10^{23} | $3.4 \times 10^{-16} \text{ m}^5$ | 2.0×10^{21} |

TABLE I. 1σ limits on $\alpha_n r_0^{n-1}$ and Λ_n in Eq. (5) from Spero, *et al.* [5] and Mitrofanov and Ponomareva [13].

FIGURES

FIG. 1. Laboratory constraints on g_p^2 and g_n^2 . The region in the g_p^2 - g_n^2 plane above and to the right of each curve is excluded at the 1σ level by the indicated experiment. The gray shading indicates the region excluded by the overlap of all present laboratory experiments, and the remaining allowed region is shown in white. The data are from Gundlach, *et al.* [2], Ramsey [16], Ritter, *et al.* [15], and Spero, *et al.* [5]. The limit from Ritter, *et al.* on g_n^2 is derived in Ref. [1].